Leonde van Beek
S2715600

**Introduction to Radio Astronomy (WBAS14001)**

**Midterm examination – Wednesday 21 December 2016, 11:05 – 12:50**

This examination contains 3 questions worth 20 points each. All questions should be answered. For full credit, all working and calculations should be shown, and full explanations given where required. Numerical answers should be in S.I. units, unless otherwise stated. Below is a list of constants and identities that may be used.

1. (a) Define (in words), what is meant by the *brightness temperature* of an object [1 point].

(b) Show that, given the Planck function,

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}, \tag{1}$$

the spectral intensity in the low-frequency regime can be expressed as,

$$B_\nu(T) = \frac{2kT}{\lambda^2} \tag{2}$$

where the symbols have their usual meaning [3 points].

(c) The moon is a thermal radio source with a brightness temperature of 213 K and an angular size of 0.5 deg on the sky. Calculate the spectral brightness, and hence the flux density of the Moon in Jy, if it were observed at 11.7 GHz [4 points].

(d) The emission and absorption of radiation through a medium of length $s$ can be described via the equation of radiative transfer,

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \epsilon_\nu, \tag{3}$$

where the symbols have their usual meaning. Using this equation, show that for a medium in thermal dynamic equilibrium (no net emission or absorption), the spectral intensity can be described by Kirchoff's law,

$$B_\nu = \frac{\epsilon_\nu}{\kappa_\nu}, \tag{4}$$

where the symbols have their usual meaning [2 points].

(e) Show also that in the case of absorption only,

$$I_\nu(s_{\text{out}}) = I_\nu(s_{\text{in}})e^{-\tau} \text{ where } \tau = \int_{s_{\text{out}}}^{s_{\text{in}}} -\kappa_\nu(s)\,ds \tag{5}$$

where the symbols have their usual meaning [3 points].

(f) Hence, show that the brightness temperature, $T_b(s)$, of the radiation at position $s$ in the medium is given by,

$$T_b(s) = T_b(0)e^{-\tau_\nu(s)} + T(1-e^{-\tau_\nu(s)}) \tag{6}$$

where $T$ is the thermodynamic temperature of the medium, and the medium is assumed to be in local thermodynamic equilibrium [5 points].

(g) If the observation described in part (c) where carried out when the sky zenith optical depth is $\tau = 0.02$, for an atmospheric temperature of 290 K and a temperature of the cosmic microwave background of 2.73 K, calculate the total brightness temperature that is observed in the direction of the Moon, for an observation at 30 degrees from zenith [2 points].

2. (a) The magnetic and electric fields that are produced by a short dipole with a constant current distribution are,

$$H_\phi = -i\frac{I\Delta l}{2\lambda}\frac{\sin\theta}{r}\left[1-\frac{1}{i\,k\,r}\right]e^{-i(\omega t - kr)}, \tag{7}$$

and,

$$E_\theta = -i\sqrt{\frac{\mu_0}{\epsilon_0}}\frac{I\Delta l}{2\lambda}\frac{\sin\theta}{r}\left[1-\frac{1}{i\,k\,r}+\frac{1}{(i\,k\,r)^2}\right]e^{-i(\omega t - kr)}, \tag{8}$$

where the symbols have their usual meaning.

Given the expressions for the magnetic and electric fields, show that, in the far-field, the time-averaged radiated power from a short dipole is given by,

$$\langle P\rangle = \frac{\pi}{3\,c\,\epsilon_0}\left(\frac{I\Delta l}{\lambda}\right)^2 \tag{9}$$

where the symbols have their usual meaning [10 points].

(b) By considering the power radiated per unit area, draw a polar diagram that shows the normalised power pattern, relative to the position of the short dipole, and label clearly the angle where the power received is attenuated by a factor of 2 [3 points].

(c) Define (in words) what is meant by the *gain* of a dipole antenna [1 point].

(d) A LOFAR low band antenna has a maximum gain of about 8 dB at a wavelength of 5 m. For this antenna, calculate the directivity and the average effective area [3 points].

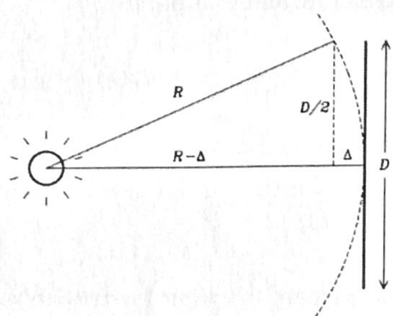
(e) Alternatively, a 13.5 m circular aperture reflector telescope, with an aperture efficiency of $\eta = 0.75$ at 5 m, can be used for your observation. How many of the dipole antennas described in part (d) are needed to provide the same effective area as this reflector telescope (you may assume that the source direction is perpendicular to the dipole antennas) [3 points].

3. (a) State three advantages and three dis-advantages that a paraboloidal reflector antenna has over a simple dipole antenna [6 points].

(b) Given the notation used in the figure below, show that the far-field distance of a paraboloidal dish antenna is given by,

$$R_{ff} = \frac{2D^2}{\lambda}, \tag{10}$$

clearly justifying any simplifying assumptions, where the symbols have their usual meaning [4 points].




(c) Calculate the far-field distance of a 12-m diameter ALMA antenna operating at 350 GHz [2 points].

(d) Define (in words) how the *current grading* of a filled aperture radio telescope is related to the resulting *far-field electric field pattern*, and how this far-field electric field pattern is related to the resulting *antenna power pattern* [2 points].

(e) Draw a schematic diagram of uniform and Gaussian current gradings, and their corresponding *power patterns*, clearly showing their relative main beam widths and side-lobe structure [4 points].

(f) Calculate the full-width at half maximum beam size of an individual 25-m Westerbork Synthesis Radio Telescope antenna at 1.4 GHz, assuming a uniform illumination [2 points].

Useful constants

Boltzmann constant $k = 1.3806488 \times 10^{-23}$ m$^2$ kg s$^{-2}$ K$^{-1}$
Speed of light in a vacuum $c = 1/\sqrt{\mu_0 \epsilon_0} = 299\,792\,458$ m s$^{-1}$
1 Jansky (Jy) $\equiv 1 \times 10^{-26}$ W m$^{-2}$ Hz$^{-1}$

Useful identities

The Taylor expansion: $e^x \approx 1 + x/1! + x^2/2! + x^3/3! + ...$
$\int_0^\pi \sin^3 \theta d\theta = 4/3$
$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$